Optimal Network Topology for a Locating System Using RSSI-based Direction Finding

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***Abstract*—Location-awareness is one of the major challenges in wireless sensor networks (WSNs) today. Received signal strength indicator (RSSI)-based localization techniques are a promising approach to provide location information in WSNs. In this paper, the performance of RSSI-based direction-of-arrival (DOA) is assessed. Therefore, the Cramér-Rao Lower Bound (CRLB) is derived in an analytical form. Since RSSI-based DOA is highly dependent on the direction of the received signal, a methodology is shown to find the optimal WSN topology evaluating the position CRLB for RSSI-based DOA measurements.**

I. INTRODUCTION

In many application domains, e.g. metering and communication, WSNs have been successfully deployed. Recently, location-awareness is more and more pushing into all aspects life and has become one of the major aspects in modern WSNs. Therefore, the need for low-cost and energy-efficient WSNs providing location information is rapidly increasing.

One of those applications is wildlife monitoring [**?**], [1]. Especially when it comes to non-intrusive observations, WSNs have a huge potential. However, the gathered sensor data is useless without any location information. To be capable to answer the bioligist's research questions, the WSNs are required to cover large areas and provide a sufficient lifetime. For that reason, being low-cost and energy-efficient are the most crucial criteria for such a network. An example for such a monitoring system with hard energy constraints is the *BATS*[1] system [2]. The general setup of a DOA-based tracking system is depicted in Fig. 1.

RSSI-based systems are an excellent choice in terms of power consumption as no complex signal processing is required within the WSN. However, RSSI-based localization techniques are said to be less precise compared to phase-based ranging. Therefore, in this paper, the performance of a localization system using RSSI-based direction finding is assessed evaluating the CRLB for direction and position estimation.

This paper is organized as follows. In Section II the principle of RSSI-based DOA estimation is introduced. Thereafter, in Section III the CRLB for RSSI-based direction finding is derived. In Section IV, the position CRLB for DOA measurements is presented. Following that, the determination of the optimal network topology applying the position CRLB is presented in Section V.

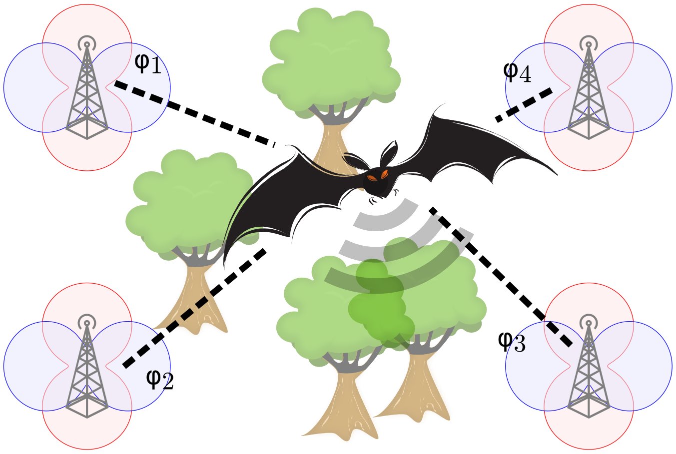

Fig. 1. Tracking bats in the wild: A WSN that estimation bat trajectories with RSSI-based DOA measurements and gathers sensor data from bats.

II. SYSTEM MODEL AND RSSI-BASED DIRECTION FINDING

The measurement principle is based on the RSSI difference for a signal received at multiple directed antennas. Assuming RSSI is measured in dB at every WSN node at each antenna, the strength of the received signal can be expressed as

$$S_a(\phi) = G_a(\phi) + P_{\mathrm{RX},a} + w_a \tag{1}$$

with P_{RX} being the signal power at the node before the antenna, $G_k(\phi)$ the gain of antenna a, and w_a an additive Gaussian noise process.

For the *BATS* application and the scope of this paper, the antenna array is restricted to two antennas comprising two half-wave dipoles at distance of $\sim \lambda/2$ and rotated by 90° towards each other. In this case the difference in received signal strength is expressed as

$$\Delta S(\phi) = S_1(\phi) - S_2(\phi) + w_1 - w_2. \tag{2}$$

When now assuming the reception of a single signal source (i.e. $P_{\mathrm{RX},1} = P_{\mathrm{RX},2}$) and the noise processes w_1 and w_2 to be uncorrelated $\Delta S(\phi)$ equation 2 reduces to

$$\Delta S(\phi) = \Delta G(\phi) + w_{1,2}, \tag{3}$$

with $\Delta G(\phi) = G_1(\phi) - G_2(\phi)$ being the gain difference function for the two considered antennas, and $w_{1,2}$ a white noise process.

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[1] Dynamic Adaptable Applications for Bats Tracking by Embedded Communicating Systems, http://www.for-bats.org/

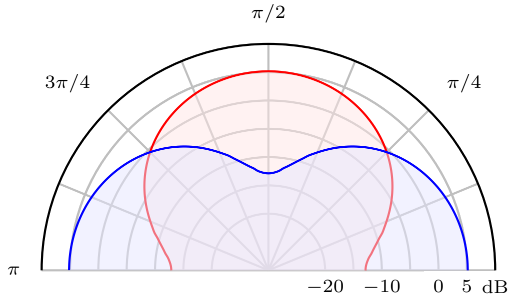


Fig. 2. The radiation patterns for both of the antennas are shown, rotated by 0° and 90° in blue and red, respectively. As the pattern are symetric, only a semicircle of the polar plot is sketched.

The antennas used for the RSSI-based DOA estimation feature a well-defined directional gain pattern and are rotated towards each other. The design process of the gain pattern and finding the optimum rotation angle between the antennas is described in-depth in [3] for more general antenna configurations, without restriction to two antennas at a distance of $\sim \lambda/2$.

The gain function for the considered half-wave dipole is given as follows

$$G = \frac{1.64}{N} \cdot \left[\frac{\cos\left(\frac{\pi}{2}\cos(\vartheta)\right)}{\sin(\vartheta)} \right]^2 \cdot E \qquad (4)$$

with a radiation pattern whose electric field E is given by

$$E = \left(\sum_{i=0}^{N-1} \sin(2\pi r_i) \right)^2 + \left(\sum_{i=0}^{N-1} \cos(2\pi r_i) \right)^2 \qquad (5)$$

and r_i being

$$\begin{aligned} r_i = [\ & (d\sin(\gamma_i) - r\sin(\phi)\sin(\vartheta))^2 \\ & + (d\cos(\gamma_i) - r\sin(\phi)\sin(\vartheta))^2 + (r\cos(\vartheta))^2\ ]^{\frac{1}{2}}. \end{aligned} \qquad (6)$$

When now assuming that the antenna consists of two dipoles $N = 2, i = [0, 1]$ and that the signal received under far-field conditions $r \gg d$ orthogonal to the dipole axes $\vartheta = \frac{1}{2}\pi$ the equations above are strapped down to this fairly simple equation

$$G(\phi) = 1.64 \cdot 2\left[\cos(2\pi d \cos(\phi))\right]^2, \qquad (7)$$

that now only depends on the rotation angle ϕ. For two antennas realizing the same gain pattern that rotated by 90° towards each other the gain difference function is expressed by

$$\Delta G(\phi) = G(\phi) - G(\phi + {}^{\pi}/_{2}). \qquad (8)$$

Fig. 2 shows the antenna pattern for the above described antenna configuration.

In Fig. 3 the gain function for the considered antennas at rotation angles of 0° and 90°, respectively, as well as the gain difference function with its derivative are depicted. As shown later evaluating the CRLB, the DOA estimation yields best performance at a high gradient of the gain difference function.

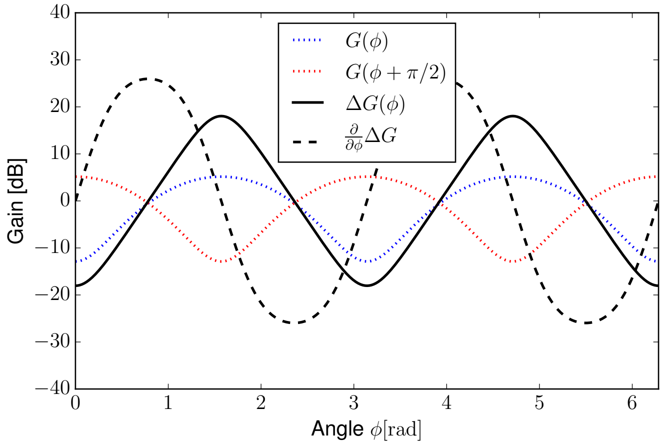


Fig. 3. The gain functions for both antenna configurations, rotated by 0° and 90° are depicted in blue and red, respectively. Also, the gain difference function (solid black) and its derivative (dashed black) are visualized.

III. THEORETICAL LIMITS IN RSSI-BASED DIRECTION FINDING

Having derived an analytical expression for the utilized direction finding antenna, now the limits of parameter estimation are considered. Assuming that a signal s depending on an unknown parameter θ is observed in white Gaussian noise (WGN), the signal model is expressed by

$$x[n] = s[n;\theta] + w[n]\ , \quad w[n] \sim \mathrm{N}(0, \sigma_{\mathrm{x}}^2). \qquad (9)$$

For $w[n]$ being a WGN process the minimum variance estimating θ is limited by the general CRLB [4]

$$\mathrm{var}(\theta) \geq \frac{\sigma_x^2}{\sum_{n=0}^{N-1} \left(\frac{\partial s[n;\theta]}{\partial \theta} \right)}. \qquad (10)$$

Assuming the noise of the RSSI difference measurement being log-normal distributed and observing RSSI difference measured in dB, the CRLB estimating DOA of the received signal yields

$$\mathrm{var}(\phi) \geq \frac{\sigma_{\Delta\mathrm{RSS}}^2}{\left(\frac{\partial \Delta G(\phi)}{\partial \phi} \right)^2}. \qquad (11)$$

As can be easily seen from Fig. 4, the variance of the DOA estimate highly depends on the direction of the received signal. Furthermore, it can be seen that the estimation variance $\mathrm{var}(\phi)$ approaches infinity for a signal direction of $\phi \approx k\frac{\pi}{2}$, which perfectly fits our expectation from Fig. 3 with the gradient of the gain difference function $\Delta G(\phi)$ being close to zero at that locations.

IV. POSITION CRAMÉR-RAO LOWER BOUND

For the sensor level, i.e. DOA estimation, a theoretical limit has been derived with the DOA CRLB in the section above. Now, the position estimation is focused. The position CRLB

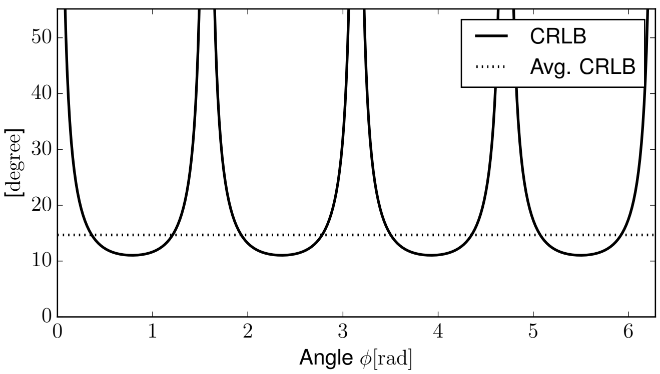


Fig. 4. CRLB for RSSI-based DOA estimation. Apparently, DOA estimation variance depends on the direction of the received signal. For a received signal direction of $\phi \approx k\frac{\pi}{2}$ the variance approaches infinity. This is quite obvious, as the gain difference function $\Delta G(\phi)$ is close to zero in that range. Therefore, network topologies, especially node orientation, matters.

gives bound on the minimum variance of the position estimator [5], [6]. When a position estimator and its covariance

$$\mathrm{var}(\hat{\boldsymbol{x}}) = E\left[(\hat{\boldsymbol{x}} - \boldsymbol{x})(\hat{\boldsymbol{x}} - \boldsymbol{x})^T\right] \tag{12}$$

is considered, the Fisher information matrix (FIM) for position estimation can be written in the alternative form [4] given by

$$F_{ij}(\boldsymbol{x}) = -E\left[\frac{\partial^2 \ln(p(\boldsymbol{\phi}|\boldsymbol{x}))}{\partial \boldsymbol{x}_i, \partial \boldsymbol{x}_j}\right], \tag{13}$$

where $\boldsymbol{\phi}$ is a vector of DOA measurements and $p(\boldsymbol{\phi}|\boldsymbol{x})$ is the probability density function (PDF) for a given user position $\boldsymbol{x} = [x, y]^T$. The joint PDF can be written as the product of the PDFs for the DOA observations ϕ_k at each sensor node

$$p(\boldsymbol{\phi}|\boldsymbol{x}) = \prod_k p(\phi_k|\boldsymbol{x}). \tag{14}$$

Then, taking the logarithm leads to

$$\ln(p(\boldsymbol{\phi}|\boldsymbol{x})) = \sum_k p(\phi_k|\boldsymbol{x}). \tag{15}$$

With measurement noise present, the relationship between position $\boldsymbol{x}$ and the measured angles $\boldsymbol{\phi}$ can be expressed as

$$\boldsymbol{\phi} = \boldsymbol{g}(\boldsymbol{x}) + \boldsymbol{w} \tag{16}$$

with

$$g_k(\boldsymbol{x}) = \tan^{-1}\frac{\Delta y_k}{\Delta x_k}, \tag{17}$$

where

$$\Delta x_k = x - x_k \quad and \quad \Delta y_k = y - y_k. \tag{18}$$

Now, assuming that $\boldsymbol{w}$ are mutually independent Gaussian random variables the likelihood function for a single DOA measurement $p(\phi_k|\boldsymbol{x})$ can be stated as

$$p(\phi_k|\boldsymbol{x}) \propto \exp\left(-\frac{1}{2\sigma_{\mathrm{DOA}}^2}\left[\phi_k - g_k(\boldsymbol{x})\right]^2\right) \tag{19}$$

which then allows to compute the FIM

$$F(\boldsymbol{x}) = \begin{bmatrix} F_{11}(\boldsymbol{x}) & F_{12}(\boldsymbol{x}) \\ F_{21}(\boldsymbol{x}) & F_{22}(\boldsymbol{x}) \end{bmatrix} \tag{20}$$

for position estimation from RSSI-based DOA measurements applying equations (13), (15), and (19) as follows

$$\begin{aligned} F_{11}(\boldsymbol{x}) &= \sum_k \frac{\Delta y_k^2}{\sigma_{\mathrm{DOA}}^2 d_k^4} \\ F_{12}(\boldsymbol{x}) &= -\sum_k \frac{\Delta x_k \cdot \Delta y_k}{\sigma_{\mathrm{DOA}}^2 d_k^4} = F_{21}(\boldsymbol{x}) \\ F_{22}(\boldsymbol{x}) &= \sum_k \frac{\Delta x_k^2}{\sigma_{\mathrm{DOA}}^2 d_k^4} \end{aligned} \tag{21}$$

with

$$d_k = \|\boldsymbol{x} - \boldsymbol{x}_k\|_2 . \tag{22}$$

Finally, inverting the FIM gives the position estimation error bound

$$\boldsymbol{\sigma}_{\mathbf{POS}}{}^2 \geq \mathrm{tr}\left(\boldsymbol{F}(\boldsymbol{x})^{-1}\right) \tag{23}$$

for DOA-based localization in a WSNs. Applying the derived position error bound, network topologies can be optimized to provide the best average localization performance for an area of interest.

V. OPTIMAL NETWORK TOPOLOGY

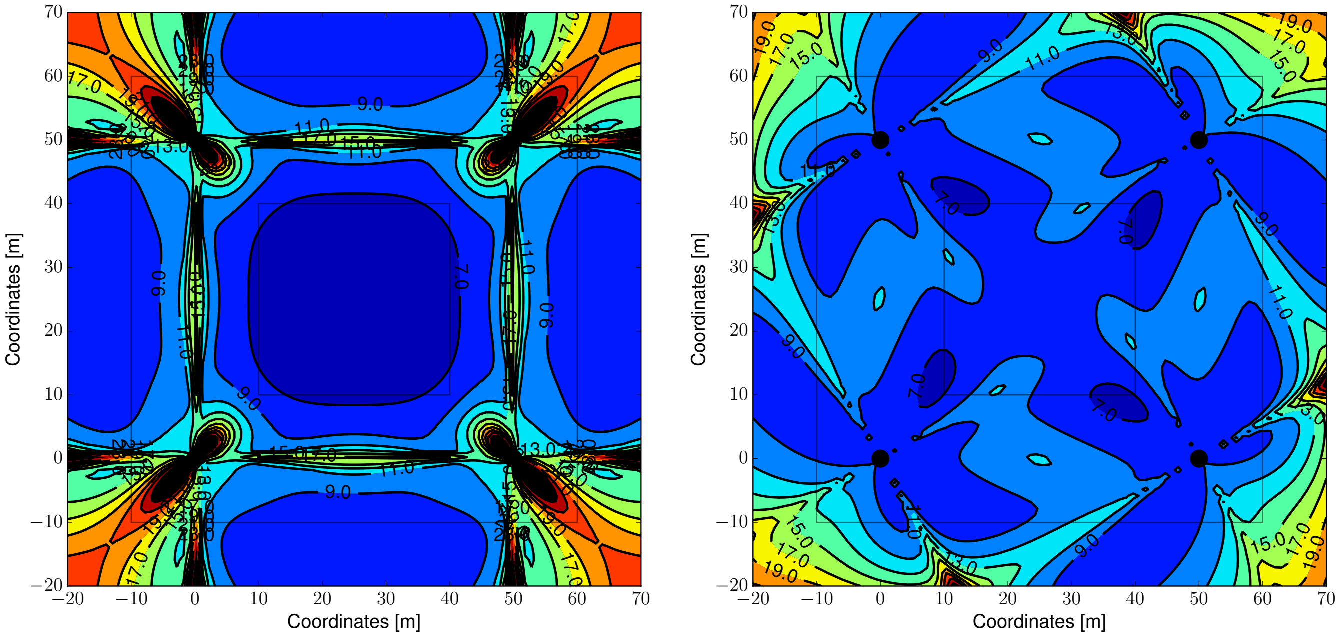
With the provided tools an optimal topology for a sensor network can be sought. As previously noted, the DOA estimation variance highly depends on the direction of the signal. It is obvious, that the position errors are dependent on signal direction and antenna orientation, too. Therefore, not only choosing the right positions of the sensor nodes is important, but also the orientation of the nodes matters.

The position CRLB for an example network has been computed for two different network configurations as shown in Table I. The nodes have been arranged in rectangular shape with a node distance of 50 m for both networks. The node orientation is 0° and 30° for the tested networks configurations 1 and 2, respectively. In both cases a log-normal distributed fading between the two receive antennas is assumed.

For both network configurations the resulting position CRLB is depicted in Fig. 5a and Fig. 5b, respectively. Apparently, the position error bound in network 1 is quite inhomogeneous. But therefore, localization performance in the center

TABLE I
SIMULATION PARAMETERS AND RESULS

Parameter		**Network 1**	**Network 2**
Node distance		50 m	
Node orientation		0°	30°
Measurement noise	$\sigma_{\Delta\mathrm{RSS}}$	5 dB	
Results		**Avg. Position Error**	
$x, y \in [+10, 40]$	$\|\boldsymbol{\sigma}_{\mathbf{POS}}\|_2$	6.290	8.393
$x, y \in [-10, 60]$	$\|\boldsymbol{\sigma}_{\mathbf{POS}}\|_2$	11.147	8.911



(a) Position CRLB for a node orientation of 0°.

(b) Position CRLB for a node orientation of 30°.

Fig. 5. CRLB for position estimation based on RSSI-based DOA measurements.

area is outstanding. For network 2 the expected position errors are more uniformly distributed, which in contrast network 1 yields a good average performance for a larger area of interest. The average position errors are computed for network 1 and 2 considering two different areas of interest $x, y \in [+10, 60]$ and $x, y \in [-10, 40]$. The simulation results are stated in Table I and are conform to the visualization in Fig. 5a and Fig. 5b.

In consideration of these results, the topology optimization of a RSSI-based locating sensor network is not only subject to the antenna characteristics, but also significantly depends on the area of interest for the objects that are to be localized. Therefore, a-priori knowledge of the spatial probability density of the tracked object can significantly help to set up a WSN with an optimal topology.

VI. CONCLUSION

In this paper the fundamentals of RSSI-based DOA have been presented. A CRLB for RSSI-based DOA estimation has been derived in an analytical form. Furthermore, a CRLB for position estimation incorporating DOA measurements has been presented. The CRLB for both, DOA and position estimation, is highly dependent on the direction of the received signal. Therefore, choosing the right network topology, especially antenna orientation, is of high importance.

However, not only antenna characteristics have a major impact on the optimal network topology, but also the area of interest matters. Utilizing the position CRLB for DOA measurements an optimal topology of the WSN can be determined minimizing the position estimation variance in the area of interest. With the tools presented in this paper a locating WSN can be arranged in an optimal way for a given antenna configuration and a certain spatial probability density of the tracked target.

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