Dynamic Multipath Mitigation Applying Unscented Kalman Filters in Local Positioning Systems

Thorsten Nowak #1, Andreas Eidloth #2

Fraunhofer Institute for Integrated Circuits IIS
Wireless Location Systems Group
RF and Microwave Design Department
Nuremberg, Germany
[1]thorsten.nowak@iis.fraunhofer.de
[2]andreas.eidloth@iis.fraunhofer.de

***Abstract*— Multipath propagation is still one of the major problems in local positioning systems today. Especially in indoor environments, the received signals are disturbed by blockages and reflections. This can lead to a large bias in the user's time-of-arrival (TOA) value. Thus multipath is the most dominant error source for positioning. In order to improve the positioning performance in multipath environments, recent multipath mitigation algorithms based upon the concept of sequential Bayesian estimation are used. The presented approach tries to overcome the multipath problem by estimating the channel dynamics, using Unscented Kalman Filters (UKF). Simulations on artificial and measured channels show the profit of the proposed estimator model.**

I. Introduction

In time-of-flight-based localization systems, the user's position can be determined by using the time-of-arrival (TOA) method. To distinguish between different users, code division multiple access (CDMA) signals are used. Major errors in the estimated position result from a bias in TOA values due to reflections.

For static multipath environments, the maximum likelihood (ML) approach performs very well, and such estimators are capable to achieve theoretical limits given by the Cramer Rao bound. Several ML approaches which address the multipath problem have also been published in the literature [1-5].

Our approach is motivated by [6] proposing joint estimation of delay and multipath coefficients. Here the channel is characterized by a time-varying tapped-delay line. As the estimator's objective are communication systems, channel approximation is limited to the sampling instance. For localization systems this restriction is not suitable.

An internal research at the Fraunhofer Institute on TOA estimation has lead to a precise method for TOA calculation using inflection points on the correlation curve [7,8], hereafter referred to as inflection-point-method (IPM). This algorithm is examined as reference for the new TOA estimator based on the Unscented Kalman Filter (UKF). In terms of performance the IPM can be compared to the Pulse Aperture Correlator [4] or the Vision Correlator [5].

In this paper, an estimator suitable for dynamic multipath channels is presented. An overview of the used time-of-flight-based localization system and the introduction of the concept of Bayesian filtering is given. Simulation results evaluating data from synthetic channel models and measurement campaigns conclude this paper.

II. Description of the Localization System

The examined localization system can be described as an inverted global navigation satellite system (GNSS). All receivers are synchronized, and the time-of-transmission (TOT) is unknown. Positions can be calculated by hyperbolic triangulation of time-difference-of-arrival (TDOA) values. Apparently errors in TOA values result in incorrect TDOA values which finally lead to a bias in the calculated position of the transmitter.

The transmitted signal is received by multiple antenna units and sampled at Nyquist rate. The sampled signal is transmitted via an optical network to the receiver units, which can be separated into hardware and software parts. The correlation of the received sequences is realized by a FPGA-based hardware component. Acquisition (which is not subject to this paper) and rough tracking of the line-of-sight (LOS) path (i.e. extracting smaller correlation windows out of the data stream) is done by the correlation hardware.

The input of the TOA estimator is a correlation window which includes 60 complex samples roughly gathered around the hardware-detected LOS path (which not necessarily has to be the real LOS path). Figure 1 describes the integration

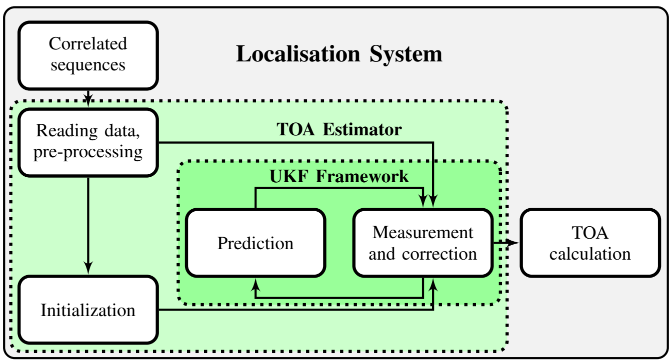

Fig. 1. Integration of the TOA estimator into the localization system.

of the TOA estimator into the localization system. After preprocessing of the correlation data, it is used for the filter initialization and the measurement in the update step of the UKF.

III. Implementation of the Estimator

The proposed TOA estimator is based on recursive Bayesian estimation [9-11]. In particular the UKF framework is used to realize the prediction and update steps of the estimator, as shown in figure 1. The UKF [12] is a non-linear extension of the original Kalman filter [13] which was designed for linear systems. As the UKF is based on the Unscented Transformation [14], this method differs essentially from Monte-Carlo methods (e.g. particle filter) as the sampling is done in a deterministic way [15,16].

A. Channel Model for Multipath Environments

It is assumed that the complex baseband-equivalent signal reaching the receiver is equal to

$$r(t) = \sum_{i=1}^{N_m} \alpha_i(t) e^{j\varphi_i(t)} \cdot s(t - \tau_i(t)) + n(t), \qquad (1)$$

where $s(t)$ is the transmitted CDMA signal, $\alpha_i(t)e^{\varphi_i(t)}$ and τ_i are the complex amplitudes and time delays, respectively, for each multipath component i. The received signal is disturbed by additive white Gaussian noise $n(t)$. As the receiver includes a matched filter, the received and sampled signal $r(k)$ is convoluted with the transmitted signal $s(k)$.

The state vector $\mathbf{m}_{i,k}$ for a single propagation path i covers delay, phase, amplitude as well as delay and phase rates.

$$\mathbf{m}_{i,k} = [\tau_{i,k} \; \varphi_{i,k} \; \alpha_{i,k} \; \dot{\tau}_{i,k} \; \dot{\varphi}_{i,k}] \qquad (2)$$

The first three parameters follow directly from the channel model above. Delay and phase rates are added to the state vector to incorporate frequency offsets between transmitter and receiver. In order to model multiple propagation paths using just one Kalman filter, the state vectors of the propagations paths are stacked.

$$\mathbf{m}_k = [\mathbf{m}_{1,k} \; \dots \; \mathbf{m}_{N_m,k}] \qquad (3)$$

B. Process and Measurement Model

Process and measurement model are central parts of the UKF framework. The process model realizes the transition from the current timestep $k-1$ to the next step k. The state prediction is a constant-velocity model.

$$\begin{aligned} \tau_{i,k}^- &= \tau_{i,k-1} + \dot{\tau}_{i,k-1} \cdot \triangle t + n_{\tau_i} \\ \varphi_{i,k}^- &= \varphi_{i,k} + \dot{\varphi}_{i,k-1} \cdot \triangle t + n_{\varphi_i} \\ \alpha_{i,k}^- &= \alpha_{i,k-1} \\ \dot{\tau}_{i,k}^- &= \dot{\tau}_{i,k-1} + n_{\dot{\tau}_i} \\ \dot{\varphi}_{i,k}^- &= \dot{\varphi}_{i,k-1} + n_{\dot{\varphi}_i} \end{aligned} \qquad (4)$$

As the process model is purely linear, the basic Kalman filter is used for the prediction of the state ahead. The prediction of the state $\mathbf{m}_k^-$ and its covariance $\mathbf{P}_k^-$ can be calculated as follows.

$$\begin{aligned} \mathbf{m}_k^- &= \mathbf{A}\mathbf{m}_{k-1} & (5) \\ \mathbf{P}_k^- &= \mathbf{A}\mathbf{P}_{k-1}\mathbf{A}^T + \mathbf{Q}_{k-1} & (6) \end{aligned}$$

Where $\mathbf{A}$ denotes the transition matrix and $\mathbf{Q}_{k-1}$ the noise of the linear process model.

Given the predicted state, the measurement model h is used to obtain the expected measurement. In an ideal case the output of the matched filter should be the autocorrelation of the transmitted signal.

$$\boldsymbol{\mu}_k = h(\mathbf{m}_k^-) \qquad (7)$$

Inside the measurement model the state vector $\mathbf{m}_k$ is again divided into sub-states $\mathbf{m}_{i,k}$ for each path. For simplicity $\mathbf{m}_{i,k}$ will be denoted as $\mathbf{m}_i$.

The presented measurement model reconstructs the expected received signal by examining the ideal autocorrelation of the transmitted signal. The transmitted signal $s(k)$ is convoluted with itself to obtain the autocorrelation C_{xx}. The autocorrelation is upsampled by the factor $f_{up} = 8$.

$$C_{xx}^8 = C_{xx} \uparrow^8 \qquad (8)$$

The estimated delay τ_i is multiplied with the interpolation factor f_{up} and rounded to the next integer.

$$\tau_{i,k}^8 = \lceil f_{up} \cdot \tau_i \rfloor \qquad (9)$$

$C_{xx}^8(\tau_i^8)$ returns the upsampled autocorrelation for the given delay τ_i. The autocorrelation is scaled and phase-shifted, according to α_i and φ_i from the state vector $\mathbf{m}_i$. The expected measurement $\boldsymbol{\mu}_i$ for one path can be calculated as follows

$$\boldsymbol{\mu}_i = \alpha_i \cdot e^{j \cdot \varphi_i} \cdot \left(C_{xx}^8(\tau_i^8) \downarrow^8 \right). \qquad (10)$$

In this realization the precision is limited to the eighth of a sample for the delay τ_i. But, on the other hand, the method can be implemented very efficiently by a lookup table. The expected measurement $\boldsymbol{\mu}$ can be calculated as the sum

$$\boldsymbol{\mu} = \sum_{i=1}^{N_m} \boldsymbol{\mu}_i \qquad (11)$$

of the expected measurements $\boldsymbol{\mu}_i$ of all individual propagation paths i. Sigma Points $\mathbf{X}_k$ are determined according to [17,18] for the predicted state $\mathbf{m}_k$ with

$$\mathbf{X}_k = [\mathbf{m}_k \cdots \mathbf{m}_k] + \sqrt{c} \left[0 \; \sqrt{\mathbf{P}_k} \; - \sqrt{\mathbf{P}_k} \right] \qquad (12)$$

where $c = n + \lambda$ and $\sqrt{\mathbf{P}_k}$ is the square root of the covariance matrix. All sigma points are propagated through the nonlinear function

$$\mathbf{Y}_k = h(\mathbf{X}_k), \qquad (13)$$

where the function h is applied to each sigma point vector of the matrix $\mathbf{X}_k$ separately. The transformed sigma points are scaled according to their weights

$$\boldsymbol{\mu}_k = \mathbf{Y}_k \mathbf{w}_m \qquad (14)$$

and the Kalman gain can be computed using the matrix form [18].

$$\mathbf{S}_k = \mathbf{Y}_k^- \mathbf{W}[\mathbf{Y}_\mathbf{k}^-]^T + \mathbf{R}_k \quad (15)$$
$$\mathbf{C}_k = \mathbf{X}_k^- \mathbf{W}[\mathbf{Y}_\mathbf{k}^-]^T \quad (16)$$
$$\mathbf{K}_k = \mathbf{C}_k \mathbf{S}_k^{-1} \quad (17)$$

The predicted state mean $\mathbf{m}_k^-$ and covariance $\mathbf{P}_k^-$ are updated applying the Kalman filter equations.

$$\mathbf{m}_k = \mathbf{m}_k^- + \mathbf{K}_k[\mathbf{y}_k - \boldsymbol{\mu}_k] \quad (18)$$
$$\mathbf{P}_k = \mathbf{P}_k^- - \mathbf{K}_k \mathbf{S}_k \mathbf{K}_k^T \quad (19)$$

The TOA value of the LOS path can be calculated directly from the estimated state by extracting the minimal delay

$$\tau_{min} = \min(\tau_1, \dots, \tau_{N_m}) \quad (20)$$

from the state vector $\mathbf{m}_k$. Triangulation of multiple TOA values leads to the user's position.

C. Dynamic Adaption of the State Space

In order to incorporate channel dynamics, the estimator has to be capable of adjusting the number of propagation paths. The filter's state space is dynamically adapted to the channel. An arbitrary number of propagation paths $N_m \in [1, N_{max}]$ can be modeled by the TOA estimator. Initially the UKF is setup with one propagation path which has not necessarily to be the LOS path. In case of changes in the dimension of the state space, the state vector as well as the transition and process noise matrices have to be resized.

The state vector $\mathbf{m}_k$ and covariance $\mathbf{P}_k$ have to be adapted to the new dimension of state space. The same is necessary for the process noise $\mathbf{Q}_k$ and the transition matrix $\mathbf{A}$. After the measurement step of the Kalman filter the expected measurement $\boldsymbol{\mu}_k$ is subtracted from the current measurement $\mathbf{y}_k$. Multiple criteria are verified on the remaining error signal

$$\mathbf{e}_k^{y,\mu} = \mathbf{y}_k - \boldsymbol{\mu}_\mathbf{k} \quad (21)$$

to determine candidates for new propagation paths. These candidates are approved in the next timestep. If a candidate passes all tests, the candidate's state vector $\mathbf{c}_{i,k}$ is initialized for the new propagation path i and appended to the state vector

$$\mathbf{m}_k \leftarrow [\mathbf{m}_k \quad \mathbf{c}_{i,k}]. \quad (22)$$

The noise and covariance matrices of the filter have to be resized accordingly.

For the reason of stability and to reduce the computation load propagation paths with delays out of range or negative amplitude are discarded. In this case sub-states are removed from the state vector. As mentioned before, the filter matrices have to be resized accordingly. Figure 5 illustrates tracking of multiple propagation paths and the channel dynamics.

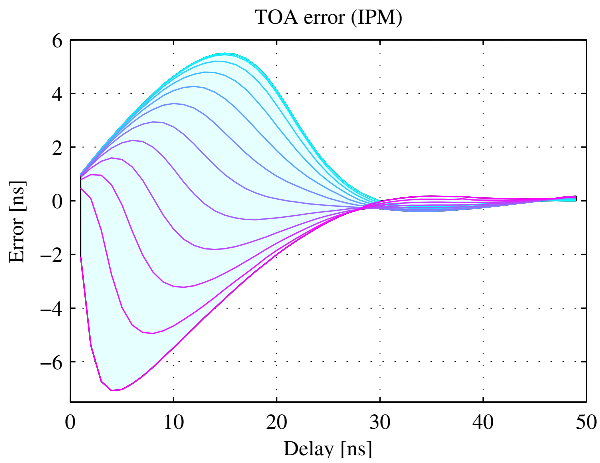

Fig. 2. TOA estimation error for the IPM. The TOA error is shown for delays up to 50 ns and phases from 0 to 165° in steps of 15°between the 2 propagation paths. Both paths are equal in amplitude.

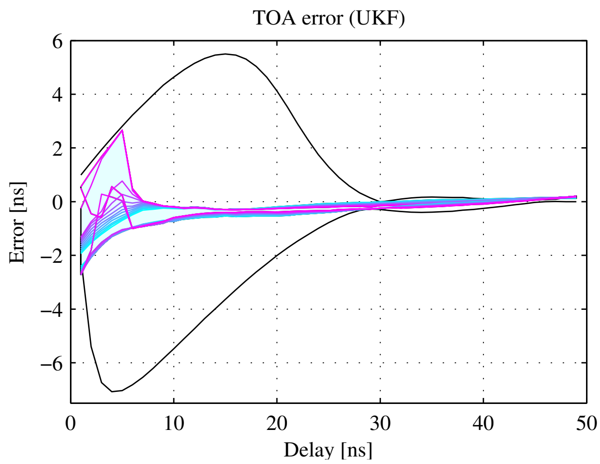

Fig. 3. The proposed UKF tracking algorithm clearly outperforms the formerly used algorithm using the inflection point method (IPM). Especially for delays between 10 and 30 ns the UKF-based TOA estimator performs excellent.

IV. Simulations

Figure 2 shows the performance of the inflection-point-method (IPM) in an artificial multipath environment with two propagation channels. The extend of the TOA error depends on the phase and delay between the two simulated propagation paths. For the IPM the error in delay estimation does not exceed 7 ns. For larger path delays (> 30 ns) no notable TOA errors exist.

The same channel is evaluated in figure 3 for the UKF. In comparison to the IPM the performance of the UKF is superior. Only minor errors exist in TOA estimation for delays larger than 7 ns. For very short path delays the modeled number of paths is reduced to one for stability reasons in some cases. Thus, a two-path-channel is modeled by only one path which leads to a larger error of approximately $e_{toa} \approx 0.5 \cdot \tau_{p_1,p_2}$. For a smaller LOS amplitude the benefit of the presented approach over the IPM is even larger.

Simulation runs of the proposed TOA estimator on data

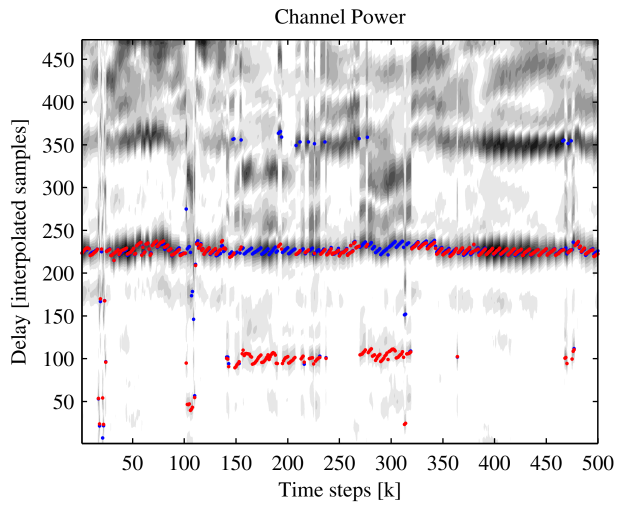

Fig. 4. Performance of the UKF tracking a moving target with up to $N_{max} = 4$ paths in a dynamic multipath environment. The figure shows the direct path estimate of the UKF (red) and the estimate from the algorithm using the inflection point (blue).

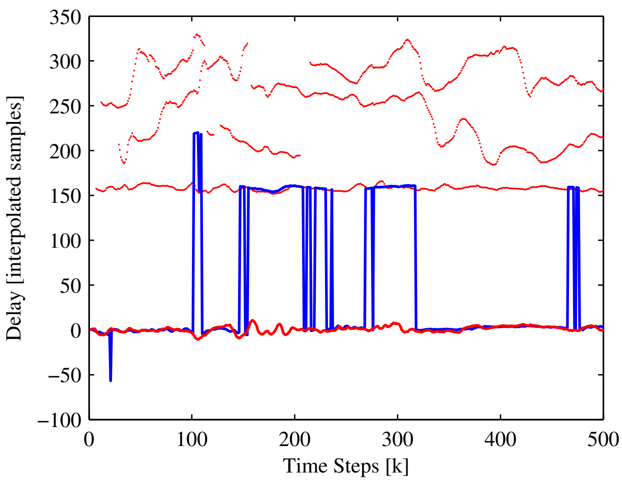

Fig. 5. Comparison of tracking performance of the UKF (red) and inflection point algorithm (blue). Dark lines show the LOS estimate. Up to three delayed propagation paths are tracked in addition to the LOS path by the UKF.

from measurement campaigns show the performance of the new algorithm in comparison to the formerly used IPM. The error of the TOA value is significantly reduced. Figure 4 depicts a real-world measured channel with strong multipath propagation. As seen in figure 5 the IPM is occasionally tracking the first or second echo, while the UKF estimator is tracking the LOS path throughout the whole measurement. Such a bias in the TOA results in a pseudo range error of 45 m.

V. Conclusion

In this paper we have proposed a new algorithm for the precise determination of the TOA value of a user's signal in case of multipath propagation. By modeling the channels multipath characteristics, the error in the TOA value due to reflections can be reduced considerably. We have demonstrated how sequential Bayesian estimation methods can be applied to the problem of multipath mitigation in localization systems. Results on synthetic and measured data confirm the benefit of the proposed TOA estimator.

References

[1] A. J. van Dierendonck, P. Fenton, and T. Ford, "Theory and Performance of Narrow Correlator Spacing in a GPS Receiver," in *Proceedings of the ION National Technical Meeting*, San Diego, California, USA, January 1992.

[2] L. R. Weill, "Achieving Theoretical Accuracy Limits for Pseudo Ranging in the Presence of Multipath," in *Proceedings of the 8th International Technical Meeting of the Satellite of the Institute of Navigation (ION GPS '95)*, Palm Springs, California, USA, 1995, pp. 1521–1530.

[3] B. R. Townsend, D. J. R. van Nee, P. C. Fenton, and K. J. V. Dierendonck, "Performance Evaluation of the Multipath Estimating Delay Lock Loop," in *Proceedings of the IEEE Position, Location and Navigation Symposium 1994*, Las Vegas, Nevada, USA, January 1994.

[4] J. Jones, P. Fenton, and B. Smith, "Performance Evaluation of the Multipath Estimating Delay Lock Loop," NovAtel, Calgary, Alberta, Canada, Tech. Rep., June 2004.

[5] P. Fenton and J. Jones, "The Theory and Performance of NovaTel Inc.'s Vision Correlator," in *Proceedings of the ION GNSS 2005*, Long Beach, California, USA, september 2005.

[6] R. Iltis, "Joint Estimation of PN Code Delay and Multipath Using the Extended Kalman Filter," *Communications, IEEE Transactions on*, vol. 38, no. 10, pp. 1677–1685, Oct 1990.

[7] G. Hofmann, "Forschungsbericht TOA-Schätzung bei Mehrwegeausbreitung," Fraunhofer Institute for Integrated Circiuts, Tech. Rep., February 2007.

[8] G. Hofmann and M. Breiling, "Vorrichtung und Verfahren zum Bestimmen eines Eintreffzeitpunktes einer Empfangsfolge," Patent, 2006, DE 10 2004 059 941 A1.

[9] Y. Bar-Shalom, X.-R. Li, and K. T., *Estimation with Applications to Tracking and Navigation*. Wiley Interscience, 2001.

[10] F. van der Heijden, R. Duin, D. de Ridder, and D. Tax, *Classification, Parameter Estimation and State Estimation – An Engineering Approach Using* MATLAB. John Wiley & Sons, Ltd., 2004.

[11] R. van der Merwe, "Sigma-Point Kalman Filters for Probabilistic Inference in Dynamic State-Space Models," Ph.D. dissertation, OGI School of Science & Engineering, Oregon Health & Science University, Portland, OR, Apr. 2004.

[12] S. J. Julier and J. K. Uhlmann, "A New Extension of the Kalman Filter to Nonlinear Systems," 1997.

[13] R. E. Kalman, "A New Approach to Linear Filtering and Prediction Problems," *Transactions of the ASME–Journal of Basic Engineering*, vol. 82, no. Series D, pp. 35–45, 1960.

[14] S. J. Julier, J. K. Uhlmann, and H. F. Durrant-Whyte, "A New Approach for Filtering Nonlinear Systems," in *American Control Conference, 1995. Proceedings of the*, vol. 3, 1995, pp. 1628–1632 vol.3.

[15] E. A. Wan and R. van der Merwe, "The Unscented Kalman Filter for Nonlinear Estimation," in *Proceedings of Symposium 2000 on Adaptive Systems for Signal Processing, Communication and Control (AS-SPCC)*. Lake Louise, Alberta, Canada: IEEE, Oct. 2000.

[16] R. van der Merwe, E. A. Wan, and S. I. Julier, "Sigma-Point Kalman Filters for Nonlinear Estimation and Sensor-Fusion: Applications to Integrated Navigation," in *Proceedings of the AIAA Guidance, Navigation & Control Conference*, 2004, pp. 16–19.

[17] S. Särkkä, T. Tamminen, A. Vehtari, and J. Lampinen, "Probabilistic Methods in Multiple Target Tracking – Review and Bibliography," Laboratory of Computational Engineering, Helsinki University of Technology, Tech. Rep. B36, February 2004, research report B36.

[18] J. Hartikainen and S. Särkkä, "Optimal filtering with Kalman Filters and Smoothers – a Manual for Matlab Toolbox EKF/UKF," February 2008, published by Department of Biomedical Engineering and Computational Science, Helsinki University of Technology.