Antenna Pattern Optimization for a RSSI-based Direction of Arrival Localization System

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Abstract
In the area of wildlife research a gaining topic is the use of wireless sensor networks to track animals for studying their social behavior [1]. Depending on the tracked species, there are extreme requirements on weight, costs or energy consumption for the transmitter mounted on the animal. For a low cost localization system the received signal strength can be used to estimate the distance. In addition, using multiple directional antennas, a direction of arrival estimation is possible. Using sensor data fusion of multiple receivers a tracking of the animal is possible. In this paper the optimization of an antenna pattern for the direction of arrival estimation based on the received signal strength in such sensor networks is shown. For this purpose, quality parameters are defined and theoretical limits such as the Cramer Rao Lower bound are compared to the simulation results. It is shown that the proposed quality parameters allow an antenna pattern optimization. The optimized antenna patterns show lower localization errors in the trajectory simulation. These results are key enablers for developing a low cost received signal strength based localization system for tracking the flying trajectory of bats.

1 Introduction
The interest in localization information is gaining more and more, even in the field of animal research. In [2] a real-time locating system (RTLS) based on Time of Arrival (ToA) measurements for animal tracking is presented, which is used to study the social behavior of bats. Due to the high synchronization requirements, such ToA systems are very costly, and not suitable for wide area coverage. In [3] a WiFi azimuth and position tracking system using fingerprinting is presented. However, such systems need a training phase to fill their database. According to the area, this may be very time consuming. An additional main requirement for wildlife monitoring in a wide area is cost-effectiveness. In Fig.1 a field strength based system with an direction of arrival estimation based on the signal strength difference is shown. For a good energy and cost-effectiveness, the number of antennas/receiver channels per direction of arrival (DoA) estimator are limited to two (Fig.1, solid and dashed pattern). Two antennas define the smallest possible number of antennas for a DoA estimation based on field strength difference. The focus of this paper is on the optimization of the antenna pattern of the receiving antenna, to get best possible localization results. Critical antenna design parameters for DoA estimation are derived. Furthermore, quality parameters to compare different antenna patterns are proposed. Section II gives an introduction to the fundamentals of field strength based DoA estimation. In Section III quality parameter are presented to compare different antenna pattern. Section IV shows the generation of antenna patterns with dipole arrangements, and Section V discusses the obtained simulation results. In Section VI this paper is concluded.

2 Fundamentals of field strength based DoA estimation
For a field strength based DoA measurement system different kinds of directional antennas could be used. In [4] a switched beam-forming antenna is employed for DoA estimation. In this work we limit a DoA estimator to two directional antennas with fixed antenna patterns. Every DoA estimator covers a DoA range from 0 to 2π. In general, the received signal strength \( P_{RX} \) from the mobile target to the receiver can be calculated by:

\[
P_{RX} = P_{TX} - L_p + G_{TX} + G_{RX}(\varphi),
\]

where \( P_{TX} \) is the emitted power of the transmitter, \( L_p \) is the pathloss between transmitter and receiver, \( G_{TX} \) is the gain of the transmitter antenna, and \( G_{RX}(\varphi) \) is the directional receiver antenna gain as a function of the DoA \( \varphi \) of the electromagnetic wave (e.g. Fig.2). In principle, the po-
position of an object in a localization system is unknown prior to the measurement. Thus, also the distance and the pathloss $L_P$ are not exactly known prior to the measurement. Consequently, measuring the signal strength difference between two differently oriented antennas is one possibility to eliminate the unknown parameter $L_P$. Using two identical antennas with different orientation, the antenna gain of the second antenna, e.g. $G_{RX,2}(\varphi)$ can be described by the gain of the first antenna $G_{RX,1}(\varphi)$:

$$G_{RX,2}(\varphi) = G_{RX,1}(\varphi + \nu), \quad (2)$$

where $\nu$ is the rotation angle between both antennas. Then, the signal strength difference $\Delta P_{RX}$ can be defined by:

$$\Delta P_{RX} = G_{RX}(\varphi) - G_{RX}(\varphi - \nu) := \Delta G_{\nu}(\varphi) \quad (3)$$

In case of a well-known antenna pattern difference function $\Delta G_{\nu}(\varphi)$, the DoA $\varphi$ can be calculated from $\Delta P_{RX}$ without any knowledge of the pathloss $L_P$. The pattern difference function only depends on the antenna pattern $G_{RX}(\varphi)$, and the constant rotation angle $\nu$ between the two antennas. Fig. 2 shows two antenna patterns with a rotation angle of $\nu = \pi/6$, and the resulting pattern difference function $\Delta G_{\pi/6}(\varphi)$ over the DoA $\varphi$. As clearly visible, multiple DoA $\varphi$ lead to identical values of $\Delta P_{RX}$, which leads to an ambiguity in the DoA estimation. Due to our demand of a 2$\pi$ coverage with only two antennas per DoA estimator, almost all pairs of non-synthetic antenna patterns present such ambiguities. We will focus on this aspect in section 3.2.

3 Criteria for the comparison of different antenna patterns

We propose two different quality parameters to evaluate and compare the suitability of pattern difference functions, i.e. the accuracy and the ambiguity.

3.1 Accuracy

We can estimate the angle estimation accuracy for a given antenna pattern difference function by means of the Cramer Rao Lower Bound (CRLB). This CRLB defines the minimum variance of an unbiased estimator. As in every measurement system noise is also present in case of our signal strength measurement. With the assumption of an additive white Gaussian noise (AWGN) channel, the CRLB for the DoA estimation can be calculated as shown in [4]. It depends on the behavior of the pattern difference function $\Delta G_{\nu}(\varphi)$, the standard deviation of the noise $\sigma_n$, and the considered DoA $\varphi$. The CRLB is given by:

$$\sigma_{\text{CRLB}}(\varphi, \nu) = \frac{1}{\sigma_n^2} \left( \frac{\partial(\Delta G_{\nu}(\varphi))}{\partial \varphi} \right)^2 := \text{CRLB} \{\Delta G_{\nu}(\varphi)\} \quad (4)$$

We are interested in the overall localization performance in a wide area. Thus, every possible DoA $\varphi$ has to be taken into account. To cover the full DoA range from 0 to 2$\pi$, the average CRLB will be used as presented in [4]:

$$\text{CRLB} \{\Delta G_{\nu}(\varphi)\} = \frac{1}{2\pi} \int_0^{2\pi} \text{CRLB} \{\Delta G_{\nu}(\varphi)\} d\varphi \quad (5)$$
Lower values of the CRLB $\{\Delta G_{\nu}(\varphi)\}$ for a specific pattern difference function $\Delta G_{\nu}(\varphi)$ give better average accuracy for the DoA estimation.

### 3.2 Ambiguity

As shown in Sec.2, the pattern difference functions show ambiguities, i.e. multiple DoA $\varphi$ fit to a certain $\Delta G_{\nu}(\varphi)$. The ambiguity $\text{AMB} \{\Delta G_{\nu}(\varphi)\}$ is defined by the number of false DoA $\tilde{\varphi}$ that show identical signal strength differences $\Delta G_{\nu}(\varphi)$ as the true DoA $\varphi$:

$$\text{AMB} \{\Delta G_{\nu}(\varphi)\} = \{\tilde{\varphi} : 0 < 2\pi \left[ \Delta G_{\nu}(\varphi) = \Delta G_{\nu}(\tilde{\varphi}) \right] \}$$

The ambiguity $\text{AMB} \{\Delta G_{\nu}(\varphi)\}$ is still a function of the DoA $\varphi$. Hence, the average ambiguity is defined as the quality index for the ambiguity:

$$\bar{\text{AMB}} \{\Delta G_{\nu}(\varphi)\} = \lim_{p \to \infty} \frac{1}{p} \sum_{i=0}^{p-1} \text{AMB} \left\{ \Delta G_{\nu}(\frac{2\pi i}{p}) \right\},$$

where $p$ is the number of discrete DoA $\varphi$ for which the ambiguity is tested. Normally, $p$ is chosen very large to get statistically relevant results. Using these two quality parameters, i.e. the average Cramer Rao Lower Bound CRLB and the average ambiguity $\bar{\text{AMB}}$, the suitability of arbitrary antenna patterns can be compared.

### 4 Generation of antenna patterns using combined half-wave dipoles

In order to get realistic antenna patterns for our later antenna design and prototype production, we have to assume certain design criteria. A very common antenna form, the dipole, is used to create realistic antenna patterns. The dipole offers advantages, i.e. the well known and described behavior, and the simple construction. In our design multiple dipoles are combined to an array, and the resulting pattern is computed. Following [5] and [6], the electrical field components of a single half-wave dipole in Z-Orientation in a spherical coordinate system where $\varphi$ is the azimuth angle, $\theta$ is the elevation angle, and $r$ is the distance to the point of observation, is given by:

$$E_{\theta}(r, \varphi, \theta) = jZ_0 \frac{I_e}{2\pi r} \cos \left( \frac{\varphi}{2} \cos \theta \right) \frac{1}{\sin \theta} e^{-j k_{iso} r},$$

where $I_e$ is the alternating current inside the dipole, and $Z_0$ describes the free wave impedance. In the far field of an arbitrary dipole alignment, the complex field components superimpose in the point of observation. To get the far field pattern of the antenna, the complex electrical field vectors of $n$ dipoles are superimposed. This implies that there is no interaction between the dipoles. This leads to:

$$E_{\text{sum}}(r, \varphi, \theta) = \sum_{i=0}^{n-1} jZ_0 \frac{I_e}{2\pi r_i} \cos \left( \frac{\varphi}{2} \cos \theta_i \right) \frac{1}{\sin \theta_i} e^{-j k_{iso} r_i},$$

Note that the distances between the transmitter and the $i$-th dipole is a function of the radius of the dipole circle $R$. The signal intensity $S$ can be calculated by the field component $E_{\text{sum}}$, and leads to the antenna gain of the dipole array, normed by the signal intensity $S_{iso}(r)$. Here $S_{iso}(r)$ is defined by an isotropic radiator, which emits the same power as the dipole array. This leads to the antenna pattern:

$$G_{n,R}(\varphi, \theta) = \lim_{r \to \infty} \frac{S(\varphi, r, \theta)}{S_{iso}(r)} = \frac{|E_{\text{sum}}|^2}{2\pi r^2}$$

Due to the two dimensional localization in our application we define the parameter $\theta = \pi/2$. Hence, the localization takes place in the azimuth-plane.

Furthermore, we limit our observations to circular arrays with a uniform distribution as shown in Fig. 3. We investigate the influence of two parameters, the number of dipoles $n$, and the radius $R$ of the circle, on which the dipoles are uniformly placed. Using this, one antenna pattern $G_R(\varphi, \theta)$ is calculated. As described in Sec. 2, the DoA estimation is realized by the field strength difference of two antennas (3). The second antenna is a rotated version of the first antenna, which gives the pattern difference function $\Delta G_{R,\nu}(\varphi)$. To get a pattern difference function $\Delta G_{R,\nu_{opt}}(\varphi)$, the optimal rotation angle $\nu_{opt}$ has to be calculated. This is the rotation angle between the two antennas, which causes the best average accuracy for the DoA estimation. It is defined by:

$$\nu_{opt} = \arg \min_{\nu \in [0;2\pi]} \{\text{CRLB} \{\Delta G_{\nu}(\varphi)\}\}$$

For every radius $R$ of the dipole arrangement, a pattern difference function $\Delta G_{\nu_{opt}}(\varphi)$ is calculated. This is the rotation angle used in all further simulation in Sec. 5. Such an optimized antenna pattern is shown in Fig. 2, assuming the optimized rotation angle $\nu = \pi/6$. It may seem trivial to find $\nu_{opt}$ for such a simple pattern. However, in other dipole arrangements more complex patterns may be generated, and finding $\nu_{opt}$ may no longer be a trivial task.

### 5 Simulation Results

For a position estimation using multiple DoAs, both parameters, i.e. the accuracy and the ambiguity of the estimation, are important. The accuracy defines the localization error if all ambiguities can be resolved. If the ambiguity of the DoA estimation is too high, the localization error can increase dramatically. This is caused by an under-determined system of equations, and the ambiguities can not be resolved anymore. To prove the theoretical considerations and proposed quality parameters for the CRLB and the $\bar{\text{AMB}}$ of the DoA estimations described in Sec. 3, we used a Monte Carlo simulation of the localization system. We computed
the quality parameters for different antenna patterns. These results are compared to the simulation results of our localization system simulation.

5.1 Antenna pattern generation and quality parameter calculation

As described in Sec. 4, different antenna patterns can be created by combining half-wave dipoles. We simulated the antenna pattern \( G_R(\varphi, \vartheta) \) for different radii \( R \) of a circular dipole arrangement. Afterwards, for every radius \( R \) of the dipole arrangement, we calculated a pattern difference function \( \Delta G_{\nu_{opt}}(\varphi) \) using the optimized rotation angle \( \nu_{opt} \) between the two antennas as described in (11). Then we derived the \( \overline{\text{AMB}} \{ \Delta G_{\nu_{opt}}(\varphi) \} \) and the \( \text{CRLB} \{ \Delta G_{\nu_{opt}}(\varphi) \} \) as a function of \( R \). The lower plot in Fig. 4 shows the results for a parameter sweep of \( 0 < R < 2\lambda \) for \( n = 3 \) dipoles. The \( \text{CRLB} \) shows multiple minima and the \( \overline{\text{AMB}} \) increases approx. at \( R \geq 0.88\lambda \). Only using the results in Fig. 4 (lower plot) it is not clear if a lower \( \text{CRLB} \), which means higher DoA accuracy, or a lower \( \overline{\text{AMB}} \), which means less DoAs are possible, result in a lower localization error.

5.2 Localization system simulation

This simulation compares the relationship between the proposed quality parameters and the position estimation error of the localization system. More precisely, the upper bound for the ambiguity in a certain scenario is investigated. For this, different realistic antenna patterns are generated as described in Sec. 5.1, and the quality parameters are calculated. The identical antenna patterns are used in the localization system simulation to get a localization error estimation. The localization error significantly depends on the receiver arrangement and system setup. In the area of GPS it is also known as dilution of precession (DOP). To take this into account a simulation environment is used, considering all the geometrical relationships. The simulation environment simulates a trajectory of the target inside an area of base stations. The target position information relative to the base station is used to generate measurement values by means of a channel model including multi-path fading and shadowing. Hereby, the antenna patterns \( G_1(\varphi), G_2(\varphi) \) and the DoA are taken into account. The localization of the simulated measurement values, and hereby the trajectory is filtered using a Bayes filter, which includes information on the antenna pattern. However, the explanation of the full operating mode of the simulation environment is beyond the scope of this paper.

For a simulation area of 200 m by 200 m, 25 receiving base stations, and a 1000 point trajectory, the average position error is shown in Fig. 4 (upper plot). The position estimation error of the trajectory and the \( \text{CRLB} \) show minimal values for similar radius \( R \) (two of them are marked with arrows). However, it also shows, when the \( \overline{\text{AMB}} \) rises over the minimum level of 12, the position estimation error is still high, even if the \( \text{CRLB} \) gets low (according fig.4 for \( R \approx 1.02\lambda \)). This can be explained by the base station arrangement. The ambiguities can not be resolved, so the position estimation error increases, even when the DoA estimation error of a single receiver decreases. For the described setup a radius of \( R \approx 0.77\lambda \) is the best possible parameter for the circular antenna array, as shown in Fig. 4, as it obtains the lowest position estimation error. However, this is a function of the density of the base stations. If the density increases, more ambiguities can be solved and the location estimation gets more accurate even in case of more ambiguities.

6 Conclusion

We have shown that the proposed quality parameters average Cramer Rao Lower bound \( \text{CRLB} \) and average ambiguity \( \overline{\text{AMB}} \) are able to compare different antenna designs.
for DoA estimation w.r.t. localization performance. The proposed quality parameters fit well to the time consuming Monte Carlo simulation results with target trajectories that take a channel model and geometrical dependencies into account. The local minima of the CRLB are in the same area as the local minima of the position estimation error in the Monte Carlo simulations. The proposed quality parameters give the antenna designer a good indication which kind of antenna pattern will result in a good localization performance. It is shown that the single parameter CRLB has no significance on its own. Thus, optimizing the CRLB does not necessarily increase the localization performance in all cases. The second parameter, i.e. the AMB has to be investigated as well, and limits the localization accuracy. Anyhow, the accuracy strongly depends on the system setup, and how it is influenced by ambiguities. This can be proven by Monte Carlo simulations in the previously identified areas of interest and saves expensive simulation time.

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